ASYMPTOTIC ANALYSIS OF LOCAL DISTURBANCES IN A PERIODIC BOUNDARY LAYER

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1. Formulation of the Problem. The boundary layer flow past small inequalities of a solid surface, all kinds of ledges, steps, bump, etc., is one of the most typical examples of a locally forced, high-Reynolds number flow of a viscous fluid. The interest in investigating such flows may be explained by the fact that the fluid motion in the vicinity of a surface is generally accompanied by high pressure gradients, drastic changes of the skin friction, and the formation of local separation zones. An approximate mathematical model of the effects resulting in such flows was obtained successfully by employing asymptotic methods to solve the Navier–Stokes equation [1-3]. It becomes apparent that the interaction between the boundary layer and the freestream is of decisive importance for the local character of a flow for the majority of situations interesting from the practical point of view.

An asymptotic theory of periodic flows in channels with weakly deformed walls was developed in [4-6]. The results obtained prove, in the first place, that local processes in unsteady flows are exceptionally complex if the undisturbed velocity profile reveals velocity reversals at some instants of time.

A plane, locally disturbed periodic laminar boundary layer flow of an incompressible fluid is considered in the present study. The boundary layer of such type is typically present in the flow past a semiinfinite flat plate, with the freestream velocity $U_\infty(1 + k \cos t)$, where t is the nondimensional time, normalized by $T_\infty(2\pi)^{-1}$ with T_∞ being the period of oscillation. We will assume that the constant t satisfies the condition $0 < k < 1$, so that the freestream velocity does not change its direction over the entire period of oscillation. The system of Cartesian coordinates has the origin at the leading edge of the plate with the abscissa pointing downstream along the plate (see Fig. 1). The coordinates of the points in the plane of the flow are denoted by $(2\pi)^{-1}U_\infty T_\infty(x, y)$ and the corresponding components of the velocity vector by $(u, v)U_\infty$. The relative change of pressure with respect to the freestream pressure is $\rho U_\infty^2 p$, where the density ρ as well as the kinematic viscosity ν are considered as constants. The Navier–Stokes equations in nondimensional variables take the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} + k\sin t = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right),\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1.1}$$

where $\mathrm{Re} = U_\infty^2 T_\infty/(2\pi\nu) \gg 1$ is the Reynolds number. Note that due to special selection of the length scale, the Strouhal number is equal to the unity.

Let us assume that the surface of the plate has a step of height h at a distance L from the leading edge and, consequently, the shape of the surface can be described as

$$y = hf(x - L),\ 0 \leqslant x < +\infty;$$
$$f(x) = 0\ (x < 0);\ f(x) = 1\ (x \geqslant 0). \tag{1.2}$$

Before imposing stricter limitations on the selection of h and L, let us consider some properties of the flow in the undisturbed boundary layer. The boundary layer solution past a smooth plate is written as $u = U(x, Y, t) + o(1)$, $v = \mathrm{Re}^{-1/2}[V(x, Y, t) + o(1)]$, $p = o(1)$, $Y = \mathrm{Re}^{1/2} y = O(1)$, $U = \partial\Psi/\partial Y$.

The functions U and V are the solution of the standard boundary-value problem for an unsteady boundary layer, where the function k sin t represents the pressure gradient, and the

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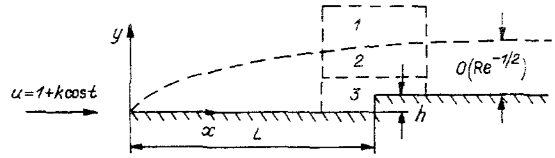

Fig. 1

velocity on the outer edge of the boundary layer equals $U(x, +\infty, t) = 1 + k \cos t$. The presented boundary-value problem is fully investigated in [7, 8]. In particular, it can be shown that in the vicinity of the leading edge of the plate the flow is quasisteady and, consequently, the skin friction on the solid surface $a(x, t) = \partial U/\partial Y\ (x, 0, t)$ can be reliably estimated as

$$a(x, t) = x^{-1/2}(1 + k \cos t)^{3/2} f_0''(0) + \ldots \qquad (x \to +0), \tag{1.3}$$

where f_0'' is the second derivative of the Blassius function.

At a larger distance from the leading edge, the boundary layer has a two-layer structure. In the near-wall region the Stokes solution is valid thus

$$a(x, t) = k \cos\left(t + \frac{\pi}{4}\right) + \ldots \qquad (x \to +\infty). \tag{1.4}$$

The estimates (1.3) and (1.4) imply the existence of a particular cross section $x = L_0$ in the boundary layer, to the left of which $(0 < x < L_0)$ the skin friction remains positive at any instant of time. For $x = L_0$, the function $a(x, t)$ becomes zero once per period, say at the instant of time $t = t_0$, $0 \le t_0 < 2\pi$. If $x > L_0$, then the skin friction becomes negative over some part of the period and the flow reversal occurs in the boundary layer.

It is assumed that $L = L_0 + Re^{-1/8}\ell$, $h = Re^{-5/8}H$ and $(\ell, H) = O(1)$. The parameter ℓ is introduced for the sake of generality and its role will be shown below.

Let us describe some analytical properties of the solution of the undisturbed boundary layer problem in the vicinity of $x = L_0$ and, incidentally, introduce a series of notations

$$\{U, \Psi\} = \sum_{n=0}^{\infty} \{U_n(Y, t), \Psi_n(Y, t)\}(x - L_0)^n \ (x \to L_0);$$

$$\Psi_n = \frac{1}{2} a_n(t) Y^2 + \frac{1}{6} b_n(t) Y^3 + \frac{1}{12} c_n(t) Y^4 + \frac{1}{20} d_n(t) Y^5 + O(Y^6)$$

$$(Y \to +0);$$

$$\{\Psi_n, U_n, a_n, b_n, c_n, d_n\} = \sum_{k=0}^{\infty} \{\Psi_{nk}, U_{nk}, a_{nk}, b_{nk}, c_{nk}, d_{nk}\}(t - t_0)^k \quad (t \to t_0).$$

According to the estimate of the instant $t = t_0$ and the distance $x = L_0$, we have $a_{00} = a_{01} = 0$ and $a_{10} < 0$. The flow velocity on the outer edge of the boundary layer is denoted by $\kappa(t) = 1 + k \cos t$, $\kappa_0 = 1 + k \cos t_0 > 0$.

Let us consider the flow within the region around the deformation. For simplicity, the consideration will be restricted to the fluid flow past a flat plate.

2. Quasisteady Regimes of Injection. Let us assume that a periodic-in-time solution of the Eq. (1.1) exists in the time interval $0 \le t < 2\pi$ near the surface (1.2). By virtue of the small extent of the region of interaction, the local Strouhal number turns out to be small. Because of this, the local flow is expected to be quasisteady at least over a large part of the period.

The first characteristic regime of the flow occurs ar $t - t_0 = O(1)$. At these instants of time, the skin friction has a nonzero value and, consequently, the velocity profile in the near-wall region of the undisturbed boundary layer depends linearly on the perpendicular coordinate. Obviously, the flow near the obstacle represents the standard flow with the three-layer scheme of interaction (triple-deck structure) [1-3] and the dependence of the solution on time is parametric.

We will show the estimates of the viscous sublayer thickness in the region of interaction and the pressure at the wall by taking into account the dependence of the above quantities on time:

$$x - L = O(\mathrm{Re}^{-3/8}\varkappa(t)^{3/2}a_0(t)^{-5/4}),$$
$$y = O(\mathrm{Re}^{-5/8}\varkappa(t)^{1/2}a_0(t)^{-3/4}),\ p = O(\mathrm{Re}^{-1/4}\varkappa(t)^{1/2}a_0(t)^{5/4}). \quad (2.1)$$

It follows from the estimates (2.1) that $t \to t_0$ the characteristic length scales of the viscous sublayer grow while the wall pressure drops. It can be shown that the momentum equation for the viscous sublayer, nonlinear for $t - t_0 = O(1)$, becomes linearized if $t \to t_0$.

The linearity condition for the velocity profile in the viscous sublayer does not hold for suffiently small values of $|t - t_0|$. Indeed, according to (2.1), the undisturbed velocity profile in the viscous sublayer can be represented in the form

$$U = \mathrm{Re}^{-1/8}\varkappa(t)^{1/2}a_0(t)^{1/4}y_1 + \mathrm{Re}^{-1/4}\frac{1}{2}b_0(t)\varkappa(t)a_0(t)^{-3/2}y_1^2 + \ldots,$$
$$y_1 = y\mathrm{Re}^{5/8}\varkappa(t)^{-1/2}a_0(t)^{3/4} = O(1).$$

If $t \to t_0$ the terms of the above expansion are of the same order for $a_0 = O(\mathrm{Re}^{-1/14})$ or $t - t_0 = O(\mathrm{Re}^{-1/28})$. In this time interval the new regime of interaction is constituted.

The second characteristic regime of interaction also appears to be quasilinear. Without providing the derivation, we will formulate the boundary-value problem for the near-wall viscous sublayer in which the variables are represented as

$$t = t_0 + \mathrm{Re}^{-1/28}t_2,\ x = L_0 + \mathrm{Re}^{-1/8}l + \mathrm{Re}^{-2/7}x_2,$$
$$y = \mathrm{Re}^{-4/7}y_2,\ (x_2,\ y_2,\ t_2) = O(1),$$
$$u = \mathrm{Re}^{-1/7}\left(\frac{1}{2}b_{00}y_2^2 + a_{02}t_2^2y_2\right) + \mathrm{Re}^{-11/56}u_2 + \ldots, \quad (2.2)$$
$$v = \mathrm{Re}^{-27/56}v_2 + \ldots,\ p = \mathrm{Re}^{-19/56}p_2 + \ldots$$

The interaction problem has the form

$$\left(\frac{1}{2}b_{00}y_2^2 + a_{02}t_2^2y_2\right)\frac{\partial u_2}{\partial x_2} + \left(b_{00}y_2 + a_{02}t_2^2\right)v_2 + \frac{\partial p_2}{\partial x_2} = \frac{\partial^2 u_2}{\partial y_2^2},\ \frac{\partial u_2}{\partial x_2} + \frac{\partial v_2}{\partial y_2} = 0, \quad (2.3)$$

where $p_2 = \frac{\varkappa_0^2}{\pi}\int\limits_{-\infty}^{+\infty}\frac{\partial A_2(s,\ t_2)}{\partial s}\frac{ds}{x_2 - s}$; $u_2 = (a_{10}l + b_{00}A_2)y_2 + a_{02}t_2^2A_2 + o(1)$ $(y_2 \to +\infty)$; $u_2 = a_{10}ly_2 + o(1)$ $(x_2 \to -\infty)$; $u_2 = -a_{02}t_2^2Hf(x_2)$, $v_2 = 0$ $(y_2 = 0)$. In deriving the boundary-value problem (2.3), the employed procedure is analogous to those given in [5, 6, 9]. Analysis of the boundary-value problem solution (2.3) for $t_2 \to 0$ shows that the asymptotic distributions (2.2) lose generality for $t_2 = O(\mathrm{Re}^{-3/112})$. The originated, third regime of interaction appears to be the most interesting since it is related to the simultaneously exhibited nonlinearity and unsteadiness of the flow.

3. Unsteady Regime of Interaction. In order to describe the third characteristic regime of interaction, we introduce new independent variables $x = L_0 + \mathrm{Re}^{-1/8}l + \mathrm{Re}^{-1/4}x_3$, $t = t_0 + \mathrm{Re}^{-1/16}t_3$, $(x_3,\ t_3) = O(1)$. In the part of the region of interaction where the flow is potential (region 1), the solution can be represented as $y_{31} = \mathrm{Re}^{1/4}y = O(1)$, $\{u,\ v,\ p\} = \{\varkappa_0,\ 0,\ 0\} + \ldots + \mathrm{Re}^{-3/8}\{u_{31}(x_3,\ y_{31},\ t_3),\ v_{31}(x_3,\ y_{31},\ t_3),\ p_{31}(x_3,\ y_{31},\ t_3)\} + \ldots$ with the expression usual for the theory of interaction

$$p_{31}(x_3,\ 0,\ t_3) = \frac{\varkappa_0}{\pi}\int\limits_{-\infty}^{+\infty}v_{31}(s,\ 0,\ t_3)\frac{ds}{s - x_3}.$$

Let us write the solution for the main part of the boundary layer:

$$u = U_{00}(Y) + \mathrm{Re}^{-1/16}t_3U_{01}(Y) + \mathrm{Re}^{-1/8}[t_3^2U_{02}(Y) +$$
$$+ lU_{10}(Y) + u_{32}(x_3,\ Y,\ t_3)] + \mathrm{Re}^{-3/16}[t_3^3U_{03}(Y) + lt_3U_{11}(Y) +$$

$$+ u_{33}(x_3, Y, t_3)] + \mathrm{Re}^{-1/4}[t_3^4 U_{04}(Y) + lt_3^2 U_{12}(Y) + x_3 U_{10}(Y) + + l^2 U_{20}(Y) + u_{34}(x_3, Y, t_3)] + \ldots, \tag{3.1}$$

$$v = \mathrm{Re}^{-3/8} v_{32}(x_3, Y, t_3) + \mathrm{Re}^{-7/16} v_{33}(x_3, Y, t_3) + + \mathrm{Re}^{-1/2}[v_{34}(x_3, Y, t_3) - \Psi_{10}(Y)] + \ldots, \quad p = \mathrm{Re}^{-3/8} p_{32}(x_3, t_3) + \ldots$$

The result of integration of the equation for the unknown coefficients of the expansion (3.1) can be written down easily. Via restrict the presentation to only the longitudinal component of the velocity vector

$$u_{32} = A_{31}(x_3, t_3) U'_{00}(Y), \quad u_{33} = A_{32}(x_3, t_3) U'_{00}(Y) + t_3 A_{31} U'_{01}(Y),$$

$$u_{34} = A_{33}(x_3, t_3) U'_{00}(Y) + t_3 A_{32} U'_{01}(Y) + \frac{1}{2} A_{31}^2 U''_{00}(Y) + \left(t_3^2 U'_{02}(Y) + lU'_{10}(Y)\right) A_{31}. \tag{3.2}$$

Here A_{31}, A_{32}, and A_{33} are unknown functions which should be found by considering the viscous sublayer (region 3). We present the solution in the viscous sublayer as

$$u = \mathrm{Re}^{-1/8}\frac{1}{2} b_{00} y_3^2 + \mathrm{Re}^{-3/16}\left[\frac{1}{2} t_3 b_{01} y_3^2 + \left(t_3^2 a_{02} + la_{10} + b_{00} A_{31}\right) y_3\right] +$$

$$+ \mathrm{Re}^{-1/4}\left[\frac{1}{4} d_{00} y_3^4 + \frac{1}{3} t_3^2 c_{01} y_3^3 + \frac{1}{2}\left(t_3^2 b_{02} + lb_{10}\right) y_3^2 + \left(t_3^3 a_{03} + lt_3 a_{11} + \right.\right.$$

$$\left.\left. + b_{00} A_{32} + t_3 b_{01} A_{31}\right) y_3 + \frac{1}{2} b_{00} A_{31}^2 + \left(t_3^2 a_{02} + la_{10}\right) A_{31} + u_3(x_3, y_3, t_3)\right] + \ldots,$$

$$v = \mathrm{Re}^{-1/2}\left[-\frac{1}{2} b_{00}\frac{\partial A_{31}}{\partial x_3} y_3^2\right] + \mathrm{Re}^{-9/16}\left[-\frac{1}{2}\left(b_{00}\frac{\partial A_{32}}{\partial x_3} + t_3 b_{01}\frac{\partial A_{31}}{\partial x_3}\right) y_3^2 - \right.$$

$$\left. - b_{00} A_{31}\frac{\partial A_{31}}{\partial x_3} y_3 - \left(t_3^2 a_{02} + la_{10}\right)\frac{\partial A_{31}}{\partial x_3} y_3 + v_3(x_3, y_3, t_3)\right] + \ldots,$$

$$p = \mathrm{Re}^{-3/8} p_3(x_3, t_3) + \ldots, \quad y_3 = y\,\mathrm{Re}^{9/16} = O(1).$$

The boundary-value problem for the viscous sublayer of the region of interaction obtained by using a standard procedure and taking into account (3.2), has the form

$$b_{00} y_3 \frac{\partial A_{31}}{\partial t_3} + \frac{\partial p_3}{\partial x_3} + \frac{1}{2} b_{00} y_3^2 \frac{\partial u_3}{\partial x_3} + b_{00} y_3 v_3 = \frac{\partial^2 u_3}{\partial y_3^2}, \quad \frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} = 0, \tag{3.3}$$

where

$$p_3 = \frac{\varkappa_0^2}{\pi}\int_{-\infty}^{+\infty}\frac{\partial A_{31}(s, t_3)}{\partial s}\frac{ds}{x_3 - s};$$

$$u_3 = o(1) \ (y_3 \to +\infty); \quad u_3 = o(1) \ (x_3 \to -\infty);$$

$$u_3 = -(A_{31} + Hf)\left[\frac{1}{2} b_{00}(A_{31} + Hf) + t_3^2 a_{02} + la_{10}\right], \quad v_3 = 0 \qquad (y_3 = 0).$$

The condition for the existence of the solution of the boundary-value problems of the type (3.3) was obtained in [10, 11]. We introduce new variables and parameters:

$$x_3 = \varkappa_0 a_{02}^{-1/6} b_{00}^{-1/3} X, \quad t_3 = \varkappa_0^{1/4} a_{02}^{-3/8} T,$$

$$A_{31} = HB(X, T), \quad \sigma = la_{10}\varkappa_0^{-1/2} a_{02}^{-1/4},$$

$$H_0 = \frac{1}{2} H b_{00}\varkappa_0^{-1/2} a_{02}^{-1/4}.$$

Let us write the condition for the existence of the solution of the problem (3.3) in the new variables in the from

$$\frac{\partial B}{\partial T} = -\gamma\frac{\partial}{\partial X}\int_{-\infty}^{X}\left\{(B(\xi, T) + f(\xi))[T^2 + \sigma + H_0(B(\xi, T) + f(\xi))] - \lambda\int_{\xi}^{+\infty}\frac{\partial^2 B(s, T)}{\partial s^2}\frac{ds}{(s - \xi)^{1/2}}\right\}\frac{d\xi}{(X - \xi)^{3/4}},$$

$$\gamma = \frac{\Gamma(5/4)}{2^{1/4}\pi}, \quad \lambda = \frac{\Gamma(3/4)}{2^{3/2}\Gamma(5/4)}. \tag{3.4}$$

Now, it is necessary to state the initial condition for (3.4). It was shown above that for $|T| \gg 1$ the flow is quasisteady, i.e., the left-hand side of the Eq. (3.4) is negligibly small comparison to the right-hand side. For $T \to -\infty$, the leading term on the right-hand side is proportional to the integral of the sum $B(X, T) + f(X)$. This yields immediately that the quasisteady condition for the flow for $T \to -\infty$ is equivalent to the initial condition for Eq. (3.4) in the form

$$B(X, T) \to -f(X) \ (T \to -\infty). \tag{3.5}$$

The problem formulated in (3.4)-(3.5) contains two nondimensional parameters. Their physical meaning is obvious; H_0 represents the effective height of the obstacle for a given region of interaction and the constant σ characterizes the distribution of the obstacle in the boundary layer. We also mention here that the combined variable $T^2 + \sigma$ can be viewed as an undisturbed skin friction upstream from the obstacle; the smaller σ, the stronger the flow reversal in the undisturbed boundary layer.

4. Well-Posedness of a Cauchy Problem and the Stability of the Solution. We mention here the analogy between (3.4) and the unsteady case of the equation based on the edge separation theory [10, 11]. It was shown in [12] that the Cauchy problem for the edge separation equation turns out to be ill-posed for suffecently general initial conditions. This brings up the question about the well-posedness of the problem in (3.4)-(3.5). Moreover, the condition of periodicity of the flow requires that the solution of the problem considered here for $T \to +\infty$ also converges to the quasisteady limit (3.5).

We consider all the above questions in the frame of the linear theory which is correct for obstacles with height $H_0 \ll 1$. (A more complete investigation of the properties of (3.4)-(3.5) will be performed separately.) Neglecting the nonlinear terms in Eq. (3.4) and carrying out the Fourier transformation in the form $B^*(\omega, T) = \int\limits_{-\infty}^{+\infty} B(X, T)\exp(-i\omega X)\,dX$, the original problem yields the ordinary differential equation solution satisfying the initial condition (3.5), has the form

$$B^* = -f^*(\omega)\left[1 - \gamma_1 (i\omega)^{5/4} |\omega| \int\limits_{-\infty}^{T} \exp[r(s)]\,ds\right],$$
$$r(s) = \frac{1}{3}\gamma_0 (i\omega)^{3/4}(s^3 - T^3) + \left[\gamma_0\sigma(i\omega)^{3/4} + \gamma_1 (i\omega)^{5/4}|\omega|\right](s - T), \tag{4.1}$$
$$\gamma_1 = 2^{-5/4}\pi^{1/2}, \ \arg\left[(i\omega)^m\right] \in (-m\pi, m\pi).$$

Here $f^*(\omega)$ is the Fourier transform of the deformation pattern. For simplicity, the function $f(X)$ can be considered sufficiently smooth.

The solution of (4.1) for $|\omega| \to \infty$ at any arbitrary finite instant of time T gives the formula $B^* = -\dfrac{\gamma_0 (T^2 + \sigma) f^*(\omega)}{\gamma_1 (i\omega)^{1/2}|\omega|}(1 + o(1))$. For large time scales this representation turns out to be nonuniform. Because of this, we take $T = |\omega|^{3/4}T_1$, where $T_1 = O(1)$. Then, for $|\omega| \to \infty$ we obtain the estimate $B^* = -f^*(\omega)\dfrac{\gamma_0 T_1^2}{\gamma_0 T_1^2 + \gamma_1 (i \operatorname{sign}\omega)^{1/2}}(1 + o(1))$. It follows directly from the previous relation that for $T_1 \to \pm\infty$ the solution (4.1) has quasisteady asymptotes.

Thus, the existence of the solution and its properties are ensured by the well-posedness of the problem (3.4)-(3.5) for infinitesimally small values of H_0. One can hope that the solution of the nonlinear equation in the limiting case possesses the analogous property, at least: in some range of variability of H_0.

Along with the question of existence of the solution of the problem (3.4)-(3.5), we will examine the stability of such a solution. For simplicity, let us assume $H_0 = 0$ and $f(x) = 0$. The equation becomes linear and uniform with the uniform initial condition. The solution of such a problem is trivial, i.e. $B = 0$ (for the undisturbed flow). Let us introduce in the stream an infinitesimally small disturbance at time $T = T_0$ which produces some distribution $B(X, T_0)$. Assuming that the further evolution of the perturbation is described by (3.4), with $H_0 = f = 0$ we get the expression for the Fourier transform of the solution

$$B^* = B^*(\omega, T_0)\exp\left\{-\frac{1}{3}\gamma_0 (i\omega)^{3/4}\left(T^3 - T_0^3\right) - \left[\gamma_0 (i\omega)^{3/4}\sigma + \gamma_1 (i\omega)^{5/4}|\omega|\right] (T - T_0)\right\}. \tag{4.2}$$

For the inverse Fourier transformation to exist, it is necessary that the function $B^*(\omega, T)$ does not grow exponentially for $\omega \to \pm\infty$. Obviously, this can be achieved only for a narrow class of initial data. In this way, the Cauchy problem with the initial data determined at a finite instant of time turns out, generally speaking, to be ill posed. Similarly to the edge separation theory, the fact that the problem is not well posed is related to the intensive growth of the amplitudes of the shortwave harmonics of the initial disturbance for the instants of time following T_0. In this case, in order to describe the evolution of the disturbances, an additional analysis of the solution for finer time and space scales [than those used while deriving (3.4)] is needed. Clearly, the initial disturbance can be adjusted in such a way that the inverse Fourier transform in Eq. (4.2) exists at any arbitrary instant of time (as example, for disturbances with a finite spectrum). Then, for $T \to +\infty$ the disturbance decays.

Let us allow the spectrum to contain harmonics with high but finite wave numbers. In the process of evolution of such disturbances, two stages can be determined. During the first stage, a rapid growth of the amplitude of the shortwave harmonics takes place. Then, at some instant of time, the perturbation decays. Clearly, the higher the wave number, the later the decay of the corresponding harmonic takes place.

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